Oscillating Slender Cone in Viscous Hypersonic Flow

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Nomenclature†

A_1, A_2, A_3, A_4	= flow perturbation coefficients
$A_x, A_\rho, A_\Delta, A'_\Delta$	= boundary-layer perturbation coefficients
C	= constant in the linear viscosity-temperature law
C_m	= pitching-moment coefficient based on cone length and cone base area
k	= $\omega l/U_\infty$, reduced frequency
K	= τM_∞, hypersonic shock angle parameter
l	= cone length
M	= Mach number
p	= pressure
Pr	= Prandtl number
Q	= density gradient factor
R_m	= functions in inviscid pressure relations
Re	= Reynolds number
T_w, T_0	= wall and stagnation temperature, respectively
u	= meridional velocity component
U_∞	= freestream velocity
x	= meridional distance from cone apex
γ	= specific heat ratio
δ_x, δ_φ	= meridional and circumferential displacement thickness, respectively
δ_ρ	= density defect
Δ	= unsteady displacement thickness
$\theta, \dot{\theta}$	= instantaneous angle-of-attack and its time rate of change, respectively
θ_c	= cone semiangle
Λ_x	= viscous interaction parameter
ξ	= axial coordinate, in terms of l, see Fig. 1
ξ_0	= ξ at the axis of oscillation
ξ_c	= $1 + \tan^2\theta_c$
ρ	= density
τ	= unperturbed shock angle
φ	= circumferential coordinate, see Fig. 1
$\bar{\chi}$	= $M^3(C/Re)^{1/2}$
$\omega/(2\pi)$	= frequency
Subscripts	
B	= at cone base
i	= local inviscid
∞	= freestream

Theme

UNSTEADY pressure distributions and stability derivatives on a slender circular cone performing oscillation in pitch in viscous hypersonic flow can be determined using the method of dynamic viscous pressure interaction developed by the present author in Ref. 1. The method takes into account the relative motion between the body surface and the boundary-layer displacement surface and can be applied to arbitrary bodies and flow conditions, provided the relevant inviscid unsteady pressure distributions and the dependence of the boundary-layer displacement on the steady flow variables are known. In the present paper, practical closed-form formulas are given for the unsteady pressure distribution and the static and dynamic pitching moment derivatives for a slender right circular cone performing low-frequency, small-amplitude oscillation in pitch around zero mean incidence and completely submerged in the weak-interaction region of laminar boundary layer. The damping-in-pitch derivative, $C_{m\dot{\theta}}$, which constitutes a sensitive indicator of the dynamic viscous interaction effects, is found to be in very good agreement with experimental results obtained for two widely different flow conditions.

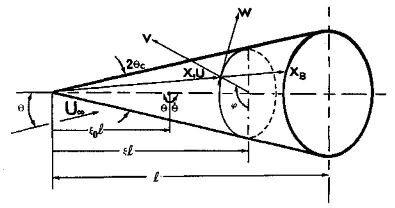

Fig. 1 Definition of certain symbols.

Contents

It was shown in Ref. 1 that if the unsteady displacement thickness Δ of a laminar boundary layer is assumed to be

$$\Delta = \bar{\Delta}[1 - \theta A_\Delta \cos\varphi - (\dot{\theta}x/\bar{u})A'_\Delta \cos\varphi] \quad (1)$$

then, in the weak interaction case, we have‡

$$\bar{\Delta} = [3/(2+Q)]\bar{\delta}_x, \quad A_\Delta = A_x + [2A_2/(2+Q)\sin\theta_c](1 - \bar{\delta}_\varphi/\bar{\Delta}) \quad (2)$$
$$A'_\Delta = [2/(2+3Q)][A_4 - QA_\Delta + (\bar{\delta}_\rho/\bar{\Delta})(A_\rho - A_4)]$$

where the various steady displacement thicknesses and their perturbation coefficients are

$$\delta_x = 2x[\bar{\chi}_i/\bar{M}_i(3)^{1/2}](d_c + 0.86/\bar{M}_i^2)(1 - \theta A_x \cos\varphi)\cos^{-1/2}\theta_c \quad (3)$$

$$\bar{\delta}_\varphi = 2x[\bar{\chi}_i/\bar{M}_i(3)^{1/2}][d_c - 0.194(\gamma - 1) + 0.472/\bar{M}_i^2]\cos^{-1/2}\theta_c \quad (4)$$

$$\delta_\rho = 2x[\bar{\chi}_i/\bar{M}_i(3)^{1/2}]\, d_c(1 - \theta A_\rho \cos\varphi)\cos^{-1/2}\theta_c \quad (5)$$

$$A_x = A_4 - A_3/2 + \{[0.894(\gamma-1) - 0.43/\bar{M}_i^2]A_1 + [0.048(\gamma-1)^2\bar{M}_i^2 + 0.367(\gamma-1) + 0.43/\bar{M}_i^2](A_2/\sin\theta_c) + 0.596(\gamma-1)(A_3 - A_4)\}\cdot[d_c + 0.86/\bar{M}_i^2]^{-1} \quad (6)$$

$$A_\rho = A_3/2 - (1/2d_c)\{[1.726/\bar{M}_i^2 - 1.788(\gamma-1)]A_1 - [1.726/\bar{M}_i^2 + 0.878(\gamma-1) + 0.093(\gamma-1)^2\bar{M}_i^2]A_2/\sin\theta_c\} \quad (7)$$

The perturbation coefficients for the meridional and circumferential velocities, pressure and density are, respectively,

$$A_1 = (U_\infty/\bar{u})\sin\theta_c, \quad A_2 = 2(U_\infty/\bar{u})\varepsilon(1 + k_a)$$
$$A_3 = -R_1/(R_0\theta_c), \quad A_4 = 2k_a\rho_\infty/[\bar{\rho}\varepsilon(1 + k_a)^2\tan\theta_c] \quad (8)$$

The functions R_m are based on the hypersonic unsteady small-disturbance theory and may be obtained from

$$R_m = -\sum_{n=0}^{6} a_{mn}\bar{K}^{-2n} \qquad (m = 0, 1, 2, 3)\ (\bar{K} > 1.04) \quad (9)$$

where coefficients a_{mn} are the same as those previously tabulated,

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Index categories: Boundary Layers and Convective Heat Transfer—Laminar; Nonsteady Aerodynamics.

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† A bar over u, ρ, M_i, δ_x, δ_ρ, and K indicates conditions at $\theta = 0$.

‡ In Eqs. (30, 32, and 106) of Ref. 1 the sign in front of the second term on the RHS of expressions for A_Δ should be +.

for $\gamma = 1.4$, following Eq. (15) in Ref. 2. For other values of γ an approximate value of the coefficients may be obtained by multiplying the tabulated values by the factor $(\tau_{air}/\tau_\gamma)^2$.

The quantities k_a, ε, and d_c are given by

$$k_a = (\gamma\varepsilon M_\infty{}^2\sin^2\theta_c)^{-1}, \qquad \varepsilon = (\gamma - 1)/(\gamma + 1) \tag{10}$$

$$d_c \approx (\gamma - 1)[0.484(T_w/T_0) + 0.145] \quad (Pr = 0.725) \tag{11}$$

Quantity Q is a density gradient factor which, for slender cones in hypersonic flow, may have values in the range 0.4–0.7. A value of $Q = 0.5$ is recommended on the basis of evidence accumulated so far.

The unsteady viscous pressure on the cone, p, is

$$\frac{p}{p_\infty\gamma\bar{K}^2} = R_0(1 + b_1\Lambda_x + b_2\Lambda_x{}^2) - \frac{\theta}{\theta_c}R_1\cos\varphi\left(1 - \frac{\bar{\Delta}}{2x}A_\Delta\cos^2\varphi\right) - ik\frac{\theta}{\theta_c}\cos\varphi\left\{(\xi R_2 - \xi_0 R_1) - \frac{\bar{\Delta}}{2x}\cos^2\varphi\left[3A_1A'_\Delta\frac{\xi R_1}{\sin\theta_c\cos\theta_c} + A_\Delta\left(\xi R_2 - \xi_0 R_1 + 2\frac{\xi R_1}{\cos\theta_c}\right)\right]\right\} \tag{12}$$

where $\Lambda_x = \bar{\chi}_\infty/(M_\infty{}^2\tan^2\theta_c\cos^{1/2}\theta_c)$. The quantities b_1 and b_2 are given in the paper as functions of γ and T_w/T_0.

The pitching-moment derivatives due to viscous pressure distribution are

$$C_{m\theta} = \frac{\partial C_m}{\partial\theta} = \frac{2\tau^2 R_1}{\theta_c\tan\theta_c}\left\{[\tfrac{1}{3}\xi_c - \tfrac{1}{2}\xi_0] - \tfrac{3}{4}A_\Delta\left(\frac{\bar{\Delta}}{x}\right)_B[\tfrac{1}{5}\xi_c - \tfrac{1}{3}\xi_0]\right\} \tag{13}$$

$$C_{m\dot\theta} = \frac{\partial C_m}{\partial(ik\theta)} = \frac{2\tau^2}{\theta_c\tan\theta_c}\left\{(\tfrac{1}{4}\xi_c - \tfrac{1}{3}\xi_0)R_2 - (\tfrac{1}{3}\xi_c - \tfrac{1}{2}\xi_0)\xi_0 R_1 - \tfrac{3}{4}\left(\frac{\bar{\Delta}}{x}\right)_B\left[\left(3\frac{A_1R_1A'_\Delta}{\sin\theta_c\cos\theta_c} + R_2A_\Delta + \frac{2A_\Delta R_1}{\cos\theta_c}\right)(\tfrac{1}{7}\xi_c - \tfrac{1}{5}\xi_0) - \xi_0R_1A_\Delta(\tfrac{1}{5}\xi_c - \tfrac{1}{3}\xi_0)\right]\right\} \tag{14}$$

Additional terms, due to circumferential skin friction, which may be important for $C_{m\theta}$ but are small for $C_{m\dot\theta}$, are given in Ref. 1. Equations (12–14) represent first-order results. A discussion of various second-order effects is given in Ref. 3 and it is shown that, at least for derivative $C_{m\dot\theta}$, only the effect of symmetrical boundary-layer interaction is significant. That effect may be included, in an approximate fashion, by artificially increasing the value of Q in the first-order calculations from 0.5 to 0.75.

The complete second-order results (with $Q = 0.5$) and the modified first-order results (with $Q = 0.75$) for derivative $C_{m\dot\theta}$ are compared in Figs. 2 and 3 with experimental results obtained at NRC-NAE and AEDC-VKF, respectively. The numerical results are somewhat different from those given in Ref. 1, in which a computational error was discovered. All experiments are for $k < 0.017$. The boundary layer was transitional for $(Re_\infty)_B > 10^7$, whereas the present calculations were all performed for laminar conditions. Inviscid results are shown for comparison. The correlation is very good despite the fact that the two sets of experiments represent two significantly different flow conditions.

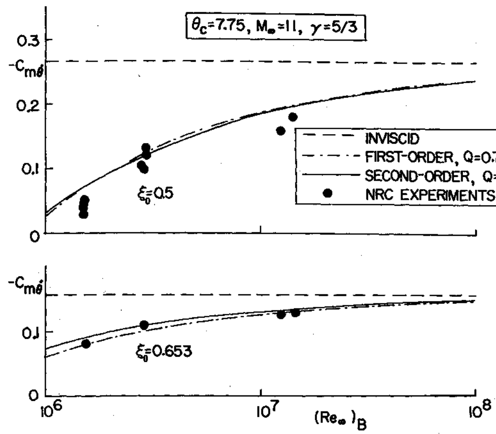

Fig. 2 Comparison of theory and experiment for a 7.75° cone at Mach 11 in helium.

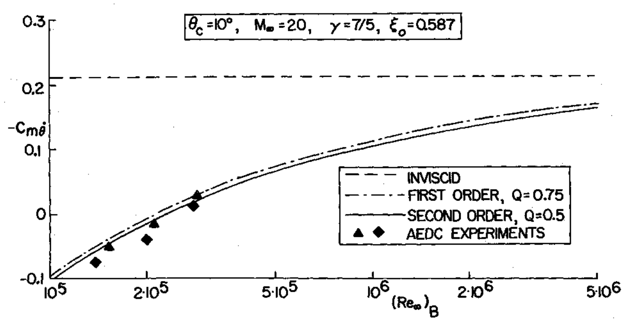

Fig. 3 Comparison of theory and experiment for a 10° cone at Mach 20 in air.

No other method to determine the effect of boundary layer on the oscillatory characteristics of a cone is known at the present time.

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